

Unit - 1

Open channel flow - I (Uniform flow).

Ref :-

FM&HM

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Introduction to open channel flow :-

* An open channel may be defined as a passage in which liquid flows with its upper surface exposed to atmosphere.

* In open channels the flow is due to gravity thus the flow conditions are greatly influenced by the slope of the channel.

* In most of the cases, the liquid is taken as water. Hence flow of water through a passage under atmospheric pressure is called flow in "open channels".

* The flow of water through pipes at atmospheric pressure (or) when the level of water in the pipe is below the top of the pipe, is also classified as open channel flow.

* Comparison between Open channel flow and pipe flow :-

S.No	Aspects	Open channel flow	Pipe flow.
1.	Cause of flow	Gravity force [Provided by sloping bottom]	The pipe is full and the flow in general takes place at the expense of hydraulic pressure.
2.	Geometry of ch.	open channels may have any shape [Triangular, rectangular, Trapezoidal, parabolic, circular etc].	Pipe generally round in c/s.

3.	Velocity distribution	The maximum velocity occurs at a little distance below the water surface. The shape of the Velocity profile is dependent upon the channel roughness.	The velocity distribution is symmetrical about the pipe axis, maximum velocity occurring at the pipe centre and the velocity at the pipe wall reducing to zero.
4.	Pitotometric Head Method	$z + y$ (pressure head/ depth) where y is the depth of flow. HGL coincides with the water surface	$C^2 + \frac{P}{\rho g}$ where P is the pressure in the pipe. HGL does not coincide with water surface.
5.	surface Roughness	Varies b/w wide limits; the hydraulic roughness varies with depth of flow.	Roughness co-efficient varies from a low value to a very high value, depending up on the material of the pipe.

* Types of flow [Classification of flow in channels].

The flow in open channel is classified into the following types:

- 1) Steady and Unsteady flow.
- 2) Uniform and Non-uniform flow
- 3) Laminar and turbulent flow.
- 4) Subcritical, critical, super Critical flows.

1) Steady and unsteady flow :-

If the flow characteristics such as depth of flow, velocity of flow, rate of flow at any point in open channel flow, don't change with respective time the flow is said to be "steady flow".

- Mathematically,

steady flow is expressed as, $\frac{\partial v}{\partial t} = 0$, $\frac{\partial \theta}{\partial t} = 0$, $\frac{\partial y}{\partial t} = 0$

* If at any point in open channel flow, the velocity of flow, depth of flow/rate of flow changes with respective time, the flow is said to be "Unsteady flow".

Mathematically,

$$\frac{\partial v}{\partial t} \neq 0$$

$$\frac{\partial \theta}{\partial t} \neq 0, \quad \frac{\partial y}{\partial t} \neq 0.$$

2. Uniform and Non-Uniform flow:-

If for a given length of the channel, the velocity of the flow, depth of flow, slope of the channel and c/s remains constant, the flow is said to be "uniform". On other hand if for a given length of channel, the velocity of flow, depth of flow or slope remains constant the flow is said to be "Non-Uniform flow".

$$\frac{\partial y}{\partial s} = 0, \quad \frac{\partial v}{\partial s} = 0 \rightarrow \text{Uniform flow.}$$

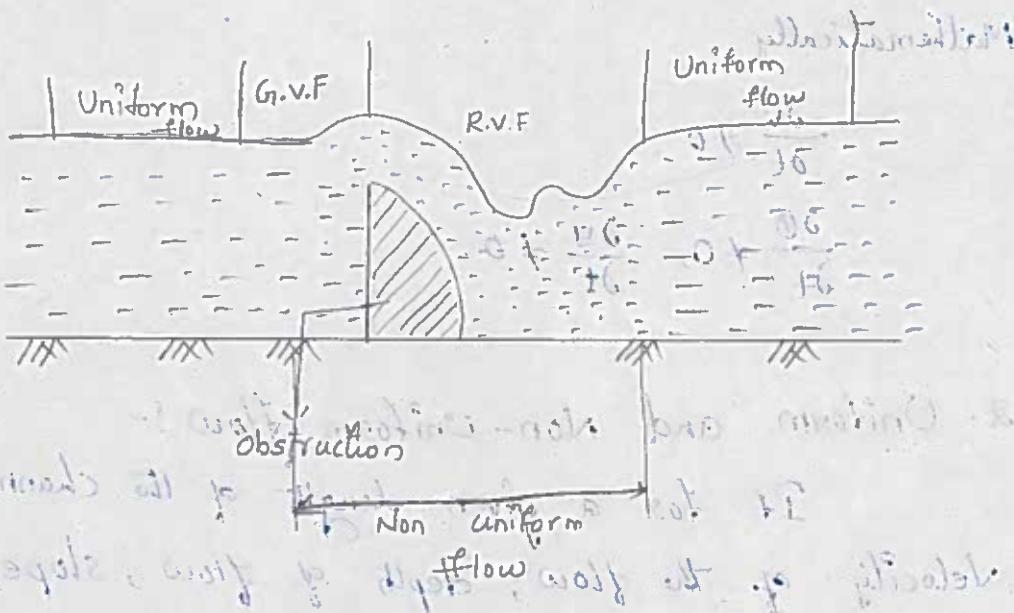
$$\frac{\partial y}{\partial s} \neq 0, \quad \frac{\partial v}{\partial s} \neq 0 \rightarrow \text{Non-uniform flow.}$$

* Non-Uniform flow in Open channels is also called Varied flow, which is classified in the following in two types:

1. Rapidly Varied flow (RVF)
2. Gradually Varied flow (GVF)

1. Rapidly Varied flow:- It is defined as that flow in which depth of flow changes about abruptly over a small length of the channel.

- Where there is any obstruction in the path of flow of water, the level of water rises above the obstruction and then falls and again rises over a small length of the channel, the flow is called "gradually varied flow".



2. Gradually Varied Flow :-

If a depth of flow in a channel changes gradually over a long length of the channel, the flow is said to be "gradually varied flow".

- It is denoted by (GVF).

3. Laminar and Turbulent flow :-

The flow in the open channel may be characterised as laminar/turbulent depending upon the value of Reynolds number, defined as,

$$Re = \frac{PVr}{\mu} = \frac{\text{Inertia forces}}{\text{Viscous forces}}$$

where,

V = Mean Velocity of flow of water

R = Hydraulic radius = A/p .

P, H = Density & Viscosity of water.

- When $Re < 500$, flow is Laminar.

- When $Re > 2000$ flow is turbulent.

- When $500 < Re < 2000$, flow is transitional.

4. Sub-critical, critical and super critical flow :-

The flow in open channel is said to be sub-critical, if its Froude number is less than 1.
→ The Froude number is defined as $\frac{V}{\sqrt{gD}} = \frac{\text{Inertia Force}}{\text{Gravitational force}}$
where,

V = Velocity of flow,

D = hydraulic depth.

1. When $f_r < 1 \rightarrow$ The flow is described as sub-critical (Tranquil or streaming).
2. When $f_r = 1 \rightarrow$ The flow is said to be in a critical state.
3. When $f_r > 1 \rightarrow$ The flow is said to be super critical (Rapid / shorting / Torrensial).

* Types of channels : The various types of channels are:

1. Natural channel
2. Artificial channel.
3. Open channel.
4. Covered / closed channel
5. Prismatic & Non-prismatic channel.

1. Natural channel :-

It is the one which has irregular sections

of varying shapes developed in a natural way.

e.g.: Rivers, streams etc.

2. Artificial channel :-

It is the one built artificially for carrying water for various purposes. They have the C/S with regular geometrical shapes.

e.g.: Rectangular channel, trapezoidal channel, parabolic etc...

3. Open channel :-
A channel without any cover at the top is known as open channel.
eg:- Irrigation canals, Rivers, streams.

4. Covered / closed channel's :-
The channel having a cover at the top is known as a covered / closed channel.
eg:- Partly filled conduits, carrying public water supply such as sewage, underground drains, Tunnels not running full of water.

5. Prismatic channel :-
A channel with unvarying c/s and the constant bottom slope is called prismatic channel.

* All the artificial channels are usually prismatic
eg:- Linear canals.

Non-prismatic channel :-

A channel with either varying c/s or the varying bottom slope is called as non-prismatic channel.

eg:- The natural channels are unlined channel.

* Geometrical properties of channel :-

a) Depth of flow (y) :- It is the vertical distance of the lowest point of the channel section from the free surface.

b) Top width (T) :- It is the width of the channel section at the free surface

c) Wetted Area (A) :- It is c/s area of the flow section of the channel.

d) Wetted perimeter :-(P) It is the length of the channel boundary in contacting with the flow of water at any section

e) Hydraulic radius / Hydraulic Mean Depth (R) :-

It is the ratio of the cross area of flow to the wetted perimeter, i.e. $R = A/p$.

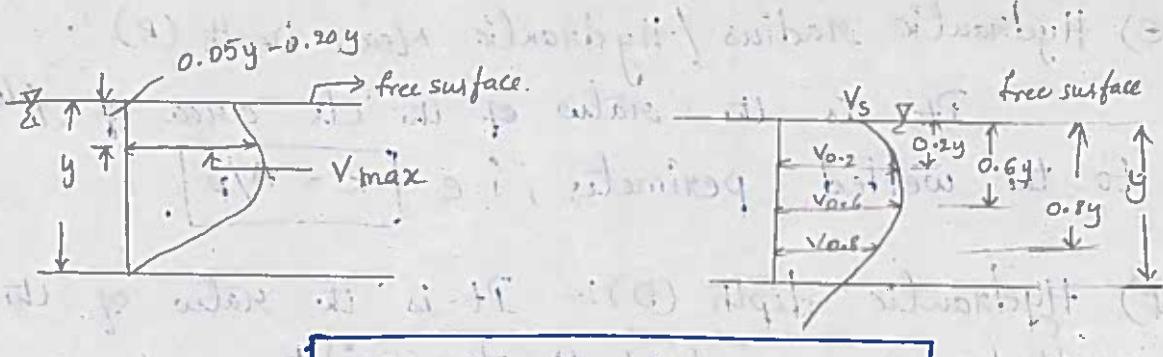
f) Hydraulic depth (D) :- It is the ratio of the wetted area and to the top width,

$$\text{i.e. } D = \frac{A}{T}$$

* Velocity distribution in a channel section :-

The velocity of flow at any channel section is not uniformly distributed. The non-uniform distributions of velocity in an open channel is due to the presence of free surface and the frictional resistance along the channel boundary.

- The Velocity distribution in a channel is measured either with help of pitot tube or a current meter.
- The Velocity distribution in a channel section depends on the various factors such as the shape of the section, the roughness of the channel and the presence of bends in the channel alignment.
- On a bend, the velocity increase greatly at a convex side due to the centrifugal action of the flow.
- Surface wind has very little effect on Velocity distribution.
- In open channel, Velocity is maximum not at free surface it is maximum at depth of 0.05y to 0.02y below free surface.
- In pipe flow the Velocity distribution is symmetrical about pipe axis, maximum Velocity occurring at pipe centre and reducing to zero at pipe wall.



$$V_{avg} = V_{0.64} = \left(\frac{V_{0.2} + V_{0.8}}{2} \right)$$



Rectangular channel Trapezoidal section

* Discharge through open channel by Chezy's Formula :-

Consider uniform flow of water over a channel with no resistance. Consider a channel as shown in the figure. If the flow is uniform, it means the velocity, depth of flow, and area of flow will be constant for a given length of the channel.

— Consider section 1-1 & 2-2

Let, L = length of channel and A

V = mean Velocity of flow of water

A = Area of flow of water.

i = slope of the bed.

P = Inletted perimeter of the c/s.

f = frictional resistance per unit velocity per unit area.

- The weight of water b/w section 1-1 & 2-2.

$w = \text{Sp. wt of water} \times \text{Volume of water}$

$$w = \frac{W}{V} \Rightarrow w \times A \times L$$

$$\Rightarrow \text{Component of } w \text{ along direction of flow} = w \sin i \\ = w \times v \sin i \\ = w \times (A \times L) \sin i \rightarrow ①$$

\Rightarrow Frictional resistance against motion of water
 $= f \times \text{surface area} \times (\text{velocity})^2$

$$\text{The value of 'n' found experimentally} = 2, \text{ and} \\ \text{Surface area} = P \times L \\ = f \times P \times L \times V^2 \rightarrow ②$$

\Rightarrow The forces acting on the water b/w 1-1 & 2-2 are

1. Component of wt of water along the direction of flows,
2. Frictional resistance against flow of water.
3. Pressure forces at sec - 1-1 & 2-2.

- As the depths of walls @ section 1-1 & section 2-2 are same, the pressure forces on these 2 sections are same and acting in opposite direction. Hence they cancel each other.

\rightarrow In case of uniform flow the velocity of flow is constant for the given length of the channels. Hence, its resultant acting in the directions of flow must be zero.

\rightarrow Resolving all forces in the direction of flow we get.

$$① = ②$$

$$w \times (A \times L) \sin i - f \times (P \times L) \times V^2 = 0$$

$$\omega \times A \times L \sin i = f \times p \times b \times v^2.$$

$$v^2 = \frac{\omega \times A \times L \sin i}{f \times p \times b}$$

$$v = \sqrt{\frac{\omega \times A \times \sin i}{f \times p}}$$

$$v = \sqrt{\frac{30}{f}} \times \sqrt{\frac{A}{p} \sin i}$$

Here

$$\sqrt{\frac{30}{f}} = C \text{ (constant of Chezy's)}$$

$$\frac{A}{p} = m = \text{hydraulic radius.}$$

for smaller values.

$$\sin i \approx \tan i \approx i$$

$$v = C \sqrt{m i}$$

$$Q = A \times v$$

$$Q = A \times C \sqrt{m i}$$

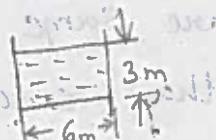
Problem

1) Find the velocity of flow and rate of flow of water through a rectangular channel of 6m wide and 3m deep, when it is running full. The channel is having bed slope as 1 in 2000. Take C = 55.

Sol Given data,

$$b = 6\text{m}, d = 3\text{m}, C = 55$$

$$\text{Bed slope} = 1 \text{ in } 2000 = \frac{1}{2000}$$



$$v = ? \quad Q = ?$$

$$v = C \sqrt{m i}$$

$$m = \frac{A}{P} \quad A = b \times d \quad P = b + 2d \\ = 6 \times 3 \quad = 6 + 2(3) \\ = 18 \text{ m}^2 \quad = 12 \text{ m}$$

$$\therefore m = 1.5 \text{ m}$$

$$V = 55 \sqrt{1.5 \times \frac{1}{2000}}$$

$$V = 1.5 \text{ m/s}$$

$$Q = A \times v = 18 \times 1.5 \Rightarrow 27 \text{ m}^3/\text{s}$$

$$Q = 27 \text{ m}^3/\text{s}$$

* Empirical formulae for the value of Chezy's Constant:-

1. Bazin Formula :-

$$C = \frac{157.6}{1.81 + \frac{k}{\sqrt{m}}} \quad \text{where } k = \text{Bazin's constant} \text{ is function of friction.}$$

where,

k = Bazin's constant and depends upon the roughness of the surface of the channel.

m = hydraulic mean depth / Radius.

S.No	Nature of channel inside surface	Value of 'k'
1	Smooth cemented / Planed wood	0.11
2	Brick / concrete / unplanned wood	0.21
3	Rubble masonry / Ashlar / poor brick work	0.83
4	Earthen channel of very good surface	1.54
5	Earthen channel of ordinary surface	2.36
6	Earthen channel of rough surface	3.17

2. Grungeillet - Kutter formula :-

$$C = \frac{23 + \frac{0.00155}{i} + \frac{1}{N}}{1 + \left(23 + \frac{0.00155}{i} \right) \frac{N}{\sqrt{m}}}$$

where,

N = Roughness co-efficient which is known as Kutter constant.

i = slope of the bed.

m = hydraulic mean depth.

3. Manning's formula :-

$$C = \frac{1}{N} m^{1/6}$$

where,

N = Manning's Constant same value of Kutter's Constant.

- This formulae are mainly used for to determine Chezy's constant.
- Value of N in Kutter's & Manning's formula

$$N = 0.010 - 0.030.$$

Problem

- 1) Find the discharge through a rectangular channel 2.5 wide, having depth of water 1.5m and bed slope as i in 2000. Take value $k = 2.36$, is Bazin's formulae.

Sol

Given,

$$b = 2.5 \text{ m}, d = 1.5 \text{ m}, k = 2.36$$

$$i = \frac{1}{2000}$$

$$Q = Ac \sqrt{mi}$$

$$A = 2.5 \times 1.5 = 37.5 \text{ m}^2$$

$$P = b + 2d \\ = 2.5 + 2(1.5)$$

$$P = 5.5 \text{ m}$$

$$m = A/P = \frac{37.5}{5.5} = 0.68 \text{ m}$$

$$C = \frac{157.6}{1.81 + \frac{k}{\sqrt{m}}} = \frac{157.6}{1.81 + \frac{2.36}{\sqrt{0.68}}} = 33.74$$

$$C = 33.74$$

$$Q = Ac \sqrt{mi} = 37.5 \times 33.74 \sqrt{0.68 \times \frac{1}{2000}}$$

$$Q = 2.83 \text{ m}^3/\text{sec}$$

* Most Economical Section of channel :-

A section of channel is said to be most economical when the cost of construction of the channel is minimum. But the cost of construction of channel depends upon the excavation and lining to keep the cost minimum, the wetted perimeter for a given discharge should be minimum, this condition is utilized for determining the dimensions of a economical sections of different form of channels.

Most economical section is also called the best section/most efficient section as the discharge, passing through a most economical section of channel for a given c/s area (A); slope of the bed (i), resistance co-efficient is max. But the discharge (Q) is given by

$$Q = AC \sqrt{mi} \quad [\because m = f/p]$$

$$Q = k \times \frac{1}{\sqrt{P}}$$

$$k = AC \sqrt{Ai}$$

$$Q = AC \sqrt{\frac{A}{P} \times i}$$

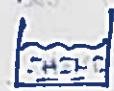
Hence the discharge (Q) will be maximum, when the wetted perimeter (P) is minimum. This condition will be used for determining the best section of a channel i.e., best dimension of a channel for a given data.

This conditions to be most economical for the following shapes of the channels, will be consider

- Rectangular channel.
- Trapezoidal channel.
- Circular channel.

* Most economical Rectangular channel:-

The Condition for most economical section that for given area the wetted perimeter should be minimum.



- Consider a rectangular channel.

Let,

b = width of channel.

d = depth of flow.

$$\text{Area of flow } (A) = bd \rightarrow ①$$

$$b = A/d$$

$$-\text{Wetted perimeter } (P) = b + 2d \rightarrow ②$$

Substitute the value of b in eqn ②.

$$P = \frac{A}{d} + 2d \rightarrow ③$$

For most economical section, P should be minimum for a given area.

$$\frac{dp}{dd} = 0$$

- Differentiating eqn ③ with respect to d and eqn same to 0.

$$\frac{d}{d(d)} (A/d + 2d) = 0$$

$$\frac{A}{d^2} + 2 = 0$$

$$A = 2d^2$$

$$b \times d = 2d^2$$

$$b = 2d$$

Hydraulic mean depth (m) = A/P .

- It is clear that rectangular channel, will be most economical, when

1. Either $b=2d$, mean width is 2 times depth of flow.

2. $m = d/2$ mean hydraulic depth is half the depth of flow.

Problem

① A rectangle channel of width 4m is having a bed slope of 1 in 1500. Find the maximum discharge through the channel. Take the value $C = 50$.

Sol Given,

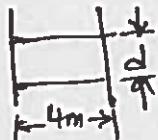
width of the channel (b) = 4m

$$d = ?$$

Bed slope (i) = $1/1500$,

$$C = 50$$

$$Q = Ac\sqrt{mi}$$



for most economical rectangular section

$$b = 2d \quad \text{(from } b = 4 \text{)} \quad \text{Retaining bottom}$$

$$d = b/2$$

$$\Rightarrow \frac{4}{2} = 2m \quad b + d =$$

\rightarrow hydraulic mean depth (m) = $d/2$

$$[\text{int. } k = \frac{2}{3} = 2]$$

$$Q = Ac\sqrt{mi}$$

$$Q = 8 \times 50 \sqrt{1 \times \frac{1}{1500}} \quad [\text{int. } k = \frac{2}{3} = 2]$$

$$Q = 10.37 \text{ m}^3/\text{sec}$$



Most economical Trapezoidal channel :-

The most economical trapezoidal channel:

The trapezoidal section of a channel will be most economical. When its perimeter is minimum

- Consider a trapezoidal section of a channel

let

b = width of channel at bottom.

d = depth of flow.

θ = angle made by the

sides with horizontal.

1) The side slope is given 1V to nH

$$\Rightarrow \text{Area of flow } (A) = \frac{Bc + Ad}{2} \times d$$
$$= \frac{b + (b + 2nd)}{2} \times d$$
$$= \frac{2b + 2nd}{2} \times d$$
$$= \frac{(b + nd) \times d}{2}$$
$$A = (b + nd) \times d \rightarrow \textcircled{1}$$

$$A/d = b + nd$$

$$b = A/d - nd \rightarrow \textcircled{2}$$

$$\text{Wetted perimeter } (P) = AB + BC + CD$$

$$= Bc + 2cD$$

$$= b + 2\sqrt{cE^2 + DE^2}$$

$$= b + 2\sqrt{d^2 + n^2 d^2}$$

$$P = b + 2d\sqrt{1+n^2} \rightarrow \textcircled{3}$$

Sub 'b' value in eqn (3)

$$P = A/d - nd + 2d\sqrt{n^2 + 1}$$

- Differentiating perimeter w.r.t 'd'

$$\frac{dp}{d(d)} = 0$$

$$\frac{d}{d(d)} [A/d - nd + 2d\sqrt{n^2 + 1}] = 0$$

$$-A/d^2 - n + 2\sqrt{n^2 + 1} = 0$$

$$-A/d^2 + n = 2\sqrt{n^2 + 1} \rightarrow \textcircled{4}$$

Sub 'A' value in eqn (4).

$$\frac{(b+nd)d}{d^2} + n = 2\sqrt{n^2 + 1}$$

$$\frac{b+nd+nd}{d} = 2\sqrt{n^2 + 1}$$

$$\frac{b+nd+d}{d} = 2\sqrt{n^2 + 1}$$

$$\boxed{\frac{b+2nd}{d} = d\sqrt{n^2 + 1}}$$

$\frac{b+2nd}{d} = \frac{1}{2}$ top width and $d\sqrt{n^2 + 1}$ = one of sloping side.

- For a trapezoidal section to be most economical which can be expressed as $\frac{1}{2}$ top width must be equal to one of the sloping side of the channel.

2) hydraulic mean depth (m) :

$$m = A/p$$

$$m = \frac{(b+nd)d}{b+2d\sqrt{n^2 + 1}} \quad \left[\frac{b+2nd}{d} = d\sqrt{n^2 + 1}, b+2d = 2d\sqrt{n^2 + 1} \right].$$

$$m = \frac{(b+nd)d}{b+b+2nd}$$

$$m = \frac{[b+nd]d}{2b+2nd}$$

$$m = \frac{[b+nd]d}{2[b+nd]}$$

$$\boxed{m = \frac{d}{2}}$$

3) Hence for a trapezoidal section to be most economical hydraulic mean depth must be equal to the depth of the flow.

4) The three sides of the trapezoidal section of a most economical section are tangential to the semi circle described on its water line.

Let

θ = Angle made by the sloping with the horizontal

O = the centre of top width AD.

Draw of DF,

Perpendicular to the AB,

$\triangle OAF$ is a right angle triangle & $\angle OAF = \theta$.

$$\text{From } \triangle OAF \Rightarrow \sin \theta = \frac{OF}{OA}$$

$$OF = OA \sin \theta$$

From $\triangle OAB$, as both are similar triangles

$$\sin \theta = \frac{d}{\sqrt{d^2 + n^2 d^2}}$$

$$\sin \theta = \frac{d}{\sqrt{1+n^2}}$$

$$\sin \theta = \frac{1}{\sqrt{1+n^2}} \quad [b(n+1) = m]$$

$$OF = OA \sin \theta$$

$$OF = d \sqrt{n^2 + 1} \times \frac{1}{\sqrt{n^2 + 1}} = m$$

$$OF = d \quad [\text{depth of flow}]$$

Hence the conditions for the most economical section are,

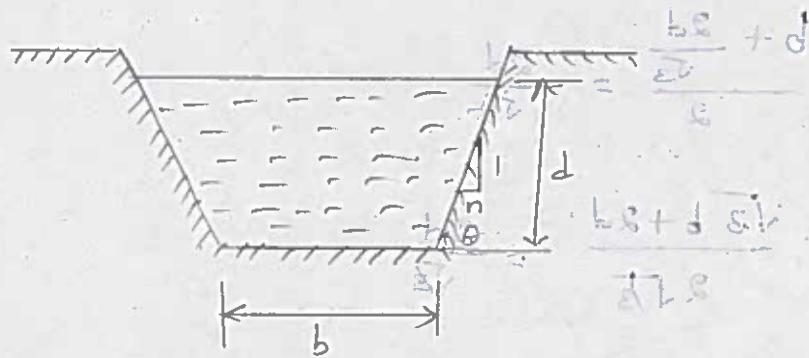
$$1) \frac{b+2nd}{2} = d \sqrt{n^2 + 1}$$

$$2) M = \frac{d}{2}$$

3) A semi circle drawn from O with radius equal to depth of flow will touch the 3 sides of the channel.

10)

Best side slope for most economical Trapezoidal section :-



$$\text{Area of flow } (A) = A(b + nd) \text{ d. s. } \downarrow$$

$$b = A/d - nd$$

$$\text{- Wetted perimeter } (P) = b + 2d \sqrt{n^2 + 1}$$

$$P = A/d - nd + 2d \sqrt{n^2 + 1}$$

$$\frac{dp}{dn} = 0$$

$$\frac{dp}{dn} \Rightarrow \frac{d}{dn} \left[A/d - nd + 2d \sqrt{n^2 + 1} \right] = 0$$

$$-d + 2nd \times \frac{1}{\sqrt{n^2 + 1}} = 0$$

$$d - 2nd \times \frac{1}{\sqrt{n^2 + 1}} = 0$$

$$2n = \sqrt{n^2 + 1}$$

Squaring on both sides,

$$4n^2 = n^2 + 1$$

$$3n^2 = 1$$

$$n = \frac{1}{\sqrt{3}}$$

$$\tan \theta = \frac{1}{n}$$

$$\tan \theta = \frac{1}{(\sqrt{3})}$$

$$\tan \theta = \sqrt{3} \quad \therefore \theta = 60^\circ$$

→ For perimeter :- For most economical trapezoidal section :-

$$\frac{b + 2nd}{2} = d \sqrt{n^2 + 1}$$

$$\text{Sub } n = \frac{1}{\sqrt{3}} \text{ in above eqn.}$$

$$\frac{b + 2 \times \frac{1}{\sqrt{3}} d}{2} = d \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}$$

$$\frac{b + \frac{2d}{\sqrt{3}}}{2} = \frac{2d}{\sqrt{3}}$$

$$\frac{\sqrt{3} b + 2d}{2\sqrt{3}} = \frac{2d}{\sqrt{3}}$$

$$\sqrt{3}b + 2d = 4d$$

$$\sqrt{3}b = 2d$$

$$b = \frac{2d}{\sqrt{3}}$$

$$\rightarrow \text{Perimeter } (P) = b + 2d \sqrt{n^2 + 1} = 9$$

$$= \frac{2d}{\sqrt{3}} + 2d \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}$$

$$= \frac{2d}{\sqrt{3}} + \frac{4d}{\sqrt{3}} = \frac{6d}{\sqrt{3}} = 2d \sqrt{3}$$

$$P = \frac{6d}{\sqrt{3}} = 2d \sqrt{3}$$

$$P = \frac{3 \times 2d}{\sqrt{3}} = 2d \sqrt{3}$$

$$\boxed{P = 3b}$$

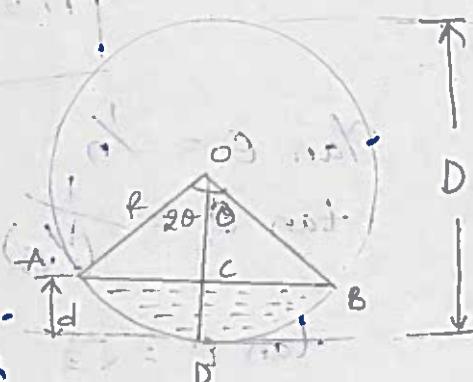
* Flow through circular channel:

Let,

d = depth of water, and

2θ = Angle subtended by water surface AB at centre in radians.

R = Radius in the channel.



i) perimeter $(P) = 2R\theta$

$$\boxed{P = 2R\theta}$$

ii) Wetted area $(A) = \text{Area of sector } ADBDA - \text{Area of triangle } CAB$:

$$2\pi = \pi R^2$$

$$2\theta = A$$

$$A = \frac{\pi R^2}{2\theta} \times 2\theta$$

$$\boxed{A = R^2 \theta}$$

$$A = \Delta OAB = \frac{1}{2} \times AB \times OC$$

$$A = \frac{1}{2} \times 2BC \times OC$$

From $\triangle OCB$

$$\sin \theta = \frac{BC}{OB}$$

$$\sin \theta = \frac{BC}{R}$$

$$BC = R \sin \theta$$

$$\cos \theta = \frac{OC}{OB}$$

$$\cos \theta = \frac{OC}{R}$$

$$OC = R \cos \theta$$

$$A = R^2 \theta - R^2 \sin \frac{2\theta}{2}$$

$$\boxed{A = R^2 \left[\theta - \sin \frac{2\theta}{2} \right]}$$

$$\begin{aligned} \text{- Area of } \triangle OAB &= \frac{1}{2} \times 2BC \times OC \\ &= \frac{1}{2} \times 2 \times R \sin \theta \times R \cos \theta \\ &= \underline{R^2 \sin \theta \cos \theta} \quad [2 \sin \theta \cos \theta = \sin 2\theta]. \end{aligned}$$

$$\text{Area of } \triangle OAB = \frac{R^2 \sin 2\theta}{2}$$

$$m = \frac{A}{P} = \frac{R^2 \left[\theta - \frac{\sin 2\theta}{2} \right]}{2R\theta}$$

$$\boxed{m = \frac{R}{2\theta} \left[\theta - \frac{\sin 2\theta}{2} \right]}$$

Problem

Find the discharge through a circular pipe at a diameter 3m, if the depth of flow of water in the pipe is 1m and the pipe is laid at a slope of 1 in 1000.

Take the value of $C = 70$.

Sol Given data,

Diameter of pipe (D) = 3m

$$R = 1.5m$$

Depth of flow (d) = 1m.

Slope (i) = 1 in 1000

$$C = 70$$

$$Q = AC\sqrt{mi}$$

from ΔACD

$$\cos \theta = \frac{OC}{OA}$$

$$\cos \theta = \frac{OC}{R} = \frac{OD - CD}{R}$$

$$\cos \theta = \frac{R-1}{R}$$

$$\cos \theta = \frac{1.5-1}{1.5}$$

$$\theta = 70.52^\circ$$

$$\theta = 70.52 \times \frac{\pi}{180} \text{ radians}$$

$$\theta = 1.23 \text{ radians}$$

$$\rightarrow \text{Area of flow} = R^2 \left[\theta - \frac{\sin 2\theta}{2} \right] \\ = (1.5)^2 \left[1.23 - \frac{\sin (2 \times 70.52^\circ)}{2} \right]$$

$$A = 2.06 \text{ m}^2$$

$$\rightarrow \text{Perimeter} = 2R\theta \Rightarrow 2 \times 1.5 \times 1.23$$

$$P = 3.69 \text{ m}$$

$$\rightarrow m = A/P$$

$$= \frac{2.06}{3.69}$$

$$m = 0.55$$

$$Q = AC\sqrt{mi}$$

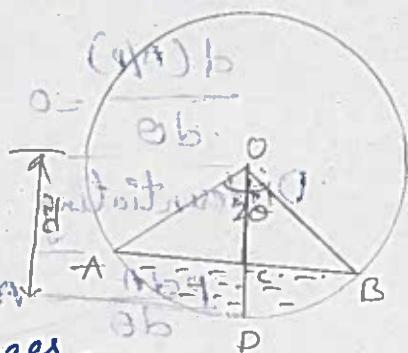
$$= 2.06 \times 70 \sqrt{0.55 \times \frac{1}{1000}}$$

$$Q = 3.38 \text{ m}^3/\text{sec}$$

≈ 0.2

(17) * Most economical circular section :-

- In case of circular section channels, the area of flow cannot be maintained constant even if the depth of flow is a constant. If the radius of circular channel of any radius, the wetted area and wetted perimeter changes, thus in case of circular channels for most economical sections to separate conditions are obtained.



They are,

- 1) Condition for max. Velocity
- 2) Condition for max. discharge

1) Condition for max. Velocity for circular sections :-

Let,

d = depth of water
 2θ = angle subtended at the centre by water surface

R = Radius of channel.

i = slope of the bed.

The Velocity of flow, according to Chezy's formula is given as

$$V = C \sqrt{mi}$$

$$V = C \sqrt{\frac{R}{P} i}$$

- The Velocity of flow through a circular channel will be maximum, when the hydraulic mean depth $m = \frac{R}{P}$ is maximum for a given value of $C \phi^i$.

- In case of circular pipe the variable is θ only.

Hence for the max. value of A/p we have condition

$$\frac{d(A/p)}{d\theta} = 0 \quad \text{--- (1)}$$

Differentiating eqn (1).

$$\frac{pdA}{d\theta} - A \frac{dp}{d\theta} = 0$$

$$\frac{p}{p_2} \frac{dA}{d\theta} - \frac{Adp}{d\theta} = 0 \rightarrow (2)$$

$$P = 2R\theta, A = R^2(\theta - \frac{\sin 2\theta}{2}).$$

$$\frac{dp}{d\theta} = 2R, \frac{dA}{d\theta} = R^2(1 - \frac{\cos 2\theta}{2})$$

$$\frac{dA}{d\theta} = R^2(1 - \cos 2\theta).$$

- Values of $P, A, \frac{dp}{d\theta}, \frac{dA}{d\theta}$ sub in eqn (2).

$$2R\theta \times R^2(1 - \cos 2\theta) - R^2(\theta - \frac{\sin 2\theta}{2}) \times 2R = 0.$$

$$2R^3\theta(1 - \cos 2\theta) - 2R^3(\theta - \frac{\sin 2\theta}{2}) = 0$$

Cancelling $2R^3$.

$$\theta(1 - \cos 2\theta) - (\theta - \frac{\sin 2\theta}{2}) = 0$$

$$\theta - \theta \cos 2\theta - \theta + \frac{\sin 2\theta}{2} = 0$$

$$-\theta \cos 2\theta + \frac{\sin 2\theta}{2} = 0$$

$$\theta \cos 2\theta = \frac{\sin 2\theta}{2}$$

$$2\theta = \frac{\sin 2\theta}{\cos 2\theta}$$

$$2\theta = \tan 2\theta.$$

The solution of the eqn by hit and trial method

$$2\theta = 257^\circ 30'$$

$$\theta = 128^\circ 45'$$

$$\theta = 128^\circ 75'$$

1) Depth of flow : OD-OC

$$OD = R, OC = R \cos \theta$$

$$d = R - R \cos \theta$$

$$d = R(1 - \cos \theta)$$

$$d = R[1 - \cos(128.75^\circ)]$$

$$d = 1.625 R, R = D/2$$

$$\boxed{d = 0.81 D}$$

The depth of flow is equal to 0.81 times of diameter of pipe.

2) Hydraulic mean depth (m) for max velocity :-

$$m = A/p = \frac{\pi R^2}{4} + \frac{\pi R^2 \sin 2\theta}{8} = 0.6R$$

$$m = \frac{R^2(1 - \frac{\sin 2\theta}{2})}{2R} = 0.5R$$

$$m = \frac{R}{2\theta} (1 - \frac{\sin 2\theta}{2})$$

$$m = \frac{R}{2 \times 2.47} [2.247 - \frac{\sin 2(128.75^\circ)}{2}]$$

$$m = 0.6R$$

$$\boxed{m = 0.30 D.}$$

2) Condition for maximum discharge for circular section.

$$Q = Ac \sqrt{mg}$$

$$Q = Ac \sqrt{\frac{A}{P}}$$

$$\left\{ Q = c \sqrt{\frac{A^3}{P}} \right\}$$

$$\frac{d}{d\theta} \left(\frac{A^3}{P} \right) = 0$$

$$\frac{P_0 A^2 \frac{dA}{d\theta} - A^3 \frac{dp}{d\theta}}{P^2} = 0$$

(1)

$$3PA^2 \frac{dA}{d\theta} - A^3 \frac{dp}{d\theta} = 0$$

$(3-1) A^2 = b$

dividing by A^2 , we get $(3-1) s = b$

$$3P \frac{dA}{d\theta} - A \frac{dp}{d\theta} = 0$$

$(3-1) s = b$

$$3 \times 2R \times R^2 (1 - \cos 2\theta) - R^2 (\theta - \frac{\sin 2\theta}{2}) \times 2R = 0$$

$$6R^3 \theta (1 - \cos 2\theta) - 2R^3 (\theta - \frac{\sin 2\theta}{2}) = 0$$

dividing by $2R^3$

$$3\theta (1 - \cos 2\theta) - (\theta - \frac{\sin 2\theta}{2}) = 0$$

$$2\theta + 3\theta \cos 2\theta + \frac{\sin 2\theta}{2} = 0$$

$$4\theta - 6\theta \cos 2\theta + \sin 2\theta = 0$$

The solution of this eqn by hit & trial method.

$$2\theta = 30^\circ$$

$$\theta = 154^\circ$$

$$\theta = 154 \times \frac{\pi}{180}$$

$$\boxed{\theta = 2.687 \text{ radian}}$$

$$d = R - R \cos \theta$$

$$= R (1 - \cos \theta)$$

$$= R (1 - \cos 154^\circ)$$

$$d = 1.89R$$

$$d = 1.89 \times 0.2$$

$$\boxed{d = 0.95D}$$

(viii) hydraulic mean depth (m) :-

$$m = A/p$$

$$m = \frac{R^2(\theta - \frac{\sin 2\theta}{2})}{2R\theta}$$

$$m = \frac{R}{2\theta} (\theta - \frac{\sin 2\theta}{2}).$$

$$m = \frac{R}{2 \times 2.687} (2.687 - \frac{\sin 308^\circ}{2}).$$

$$\boxed{m = 0.573 R}$$

$$m = 0.573 \times 0.6$$

$$\boxed{m = 0.29 D}$$

i) Condition for max. Velocity

$$d = 0.81m, m = 0.30 D.$$

ii) Condition for max. discharge

$$d = 0.95 D$$

$$m = 0.29 D.$$

Problem

The rate of flow of water through a circular channel of dia 0.6m is 150 lit/sec. Find the slope of the bed of the channel for max. Velocity.

Take $C = 60$.

Sol Given data,

$$d = 0.6m, \gamma = 0.3m$$

$$Q = 150 \text{ lit/sec} \Rightarrow m^3/\text{sec}$$

$$C = 60.$$

$$\theta = ?$$

For max Velocity conditions.

$$Q = AC\sqrt{m}$$

$$m = A/p$$

$$m = 0.30D$$

$$= 0.3 \times 0.6$$

$$= 0.18 \text{ m.}$$

$$P = 2R\theta$$

$$\theta = 128.75^\circ$$

$$= 128.75 \times \frac{\pi}{180}$$

$$P = 2.247 \text{ radians.}$$

$$P = 2 \times 0.3 \times 2.247$$

$$\boxed{P = 1.34 \text{ m.}}$$

$$A = m \times P$$

$$= 0.18 \times 1.34$$

$$\boxed{A = 0.242 \text{ m}^2}$$

$$Q = AC \sqrt{mi}$$

$$0.15 = 0.242 \times 60 \sqrt{0.18 \times i}$$

$$\boxed{i = \frac{1}{1686.6}}$$

* The factors affecting manning's Co-efficient (n).

*-1) Cross - sectional Geometry

2. Boundary roughness

3. Vegetation channel.

4. Channel Alignment

5. Obstructions

6. Size & shape of channel.

7. Seasonal change

8. Suspended material.

* Specific Energy & specific energy curve:-

The total energy of a flowing liquid per unit wt is given by

$$T.E = z + h + \frac{v^2}{2g}$$

Where,

z = ht. of the bottom of channel above datum,

h = depth of liquid

v = mean velocity of flow

If the channel bottom is taken as datum then total energy per wt of liquid will be

$$E = h + \frac{v^2}{2g}$$

This is known as specific energy.

* Hence Specific energy of a flowing liquid is defined as energy per unit wt of liquid with respect to the bottom of the channel.

- Specific energy curve

It is defined as the curve which shows the variation of specific energy of with the depth of flow.

$$E = h + \frac{v^2}{2g}$$

$$E = E_p + E_k$$

where

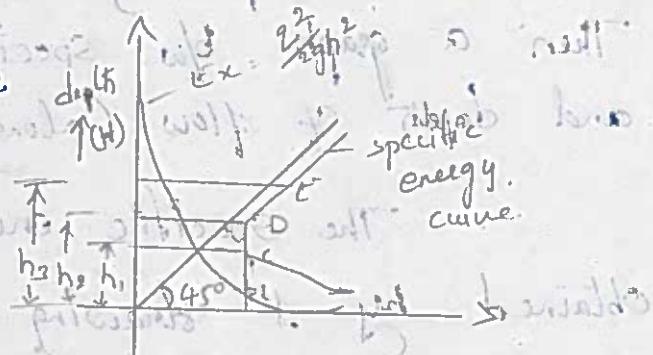
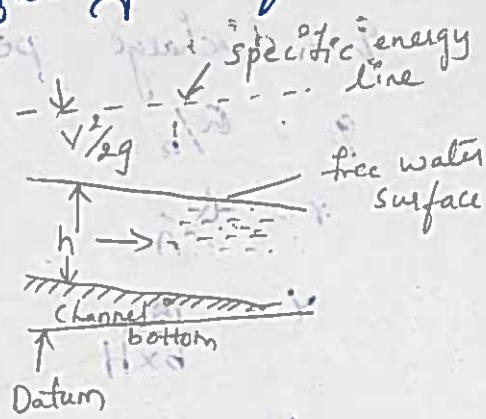
E_p = potential energy of flow $\rightarrow h$.

E_k = kinetic energy of flow $\rightarrow \frac{v^2}{2g}$

- Consider a rectangular channel

let

Q = discharge through channel.



b = width of the channel.

h = depth of flow.

q = discharge per unit width.

$$q = Q/b$$

$$r = Q/A$$

$$Y = \frac{Q}{b \times h}$$

$$V = \frac{q}{h}$$

$$\epsilon = h + \frac{V^2}{2g}$$

$$\boxed{\epsilon = h + \frac{q^2}{2gh^2}}$$

$$\epsilon = \epsilon_p + \epsilon_k$$

The above eqn gives the variation of specific energy with the depth of flow. Hence, for a given discharge, for different values of depth of flow the corresponding values of ϵ may be obtained. Then a graph b/w specific energy (ϵ along $X-X$ axis) and depth of flow (along $Y-Y$ axis) may be plotted.

The specific energy curve may also be obtained by 1st drawing a curve for potential energy ($\epsilon_p = h$) which will be a straight line passing through origin making an angle of 45° with the x -axis then drawing another curve for kinetic energy ($\epsilon_k = \frac{q^2}{2gh^2}$) which will be a parabola. Combining these two curves we can obtain the specific curve ACB denotes the specific energy curve.

16) * Critical depth :-

It is defined as that depth of flow of water at which the specific energy is minimum. The curve ACB is a specific energy curve and point 'c' corresponds to the minimum specific energy. The depth of flow of water at 'c' is known as "Critical depth".

The mathematical expression for (h_c) is obtained by differentiating the specific energy eqn with respective depth of flow and equating with respect to zero.

$$\frac{de}{dh} = 0$$

$$\frac{d}{dh} \left(h + \frac{V^2}{2gh^2} \right) = 0$$

$$1 + \frac{V^2}{2g} \left(-\frac{2}{h^3} \right) = 0$$

$$1 - \frac{V^2}{gh^3} = 0$$

$$gh^3 = V^2$$

$$h^3 = \frac{V^2}{g}$$

$$h = \left(\frac{V^2}{g} \right)^{1/3}$$

$$h_c = \left(\frac{V^2}{g} \right)^{1/3}$$

* Critical velocity :-

The velocity of the flow at the critical depth is known as Critical Velocity.

The Mathematical expression for V_c is obtained from the eqn.

$$h_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$$

$$h_c^3 = \frac{q^2}{g}$$

$$q^2 = g h_c^3$$

$$q = \sqrt[3]{gh_c^3}$$

$$q = \frac{A \times v}{b}$$

$$A \times v = b \times h_c \times v_c$$

$$q = \frac{b \times h_c \times v_c}{b} = h_c \times v_c$$

$$q = h_c \times v_c$$

$$q = h_c \times v_c$$

$$q h_c^3 = h_c^2 \times v_c^2$$

$$q h_c = v_c^2$$

$$v_c = \sqrt{q h_c}$$

* Minimum specific energy at critical depth :-

Specific energy is given by

$$E = h + \frac{q^2}{2gh^2}$$

When specific energy is minimum and depth of flow is critical in above eqn becomes.

$$E_{min} = h_c + \frac{q^2}{2gh_c^2} \rightarrow ①$$

$$h_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$$

$$\frac{q^2}{g} = h_c^3$$

$$\text{Sub } \frac{q^2}{g} = h_c^3 \text{ in eq } ①$$

$$E_{min} = h_c + \frac{h_c^3}{2h_c^2}$$

$$E_{min} = h_c + \frac{h_c}{2} \Rightarrow$$

$$E_{min} = \frac{3}{2} h_c$$

19)

Problem

Find the specific energy of flowing water through a rectangular channel of width 5m when the $Q = 10 \text{ m}^3/\text{sec}$ depth of water is 3m.

Given data, (i) Right岸に水が満たされた渠の幅

$$b = 5 \text{ m}$$

$$d = 3 \text{ m (h)}$$

$$Q = 10 \text{ m}^3/\text{sec}$$

$$\epsilon = h + \frac{V^2}{2g}$$

$$Q = Av \Rightarrow V = \frac{Q}{A}$$

$$= \frac{10}{5 \times 3}$$

$$V = 0.667 \text{ m/sec}$$

$$\epsilon = 3 + \frac{(0.667)^2}{2 \times 9.81}$$

$$\epsilon = 3.02 \text{ m}$$

* Critical flow :-
It is defined as that flow at which the specific energy is minimum or the flow corresponding to critical depth is defined as 'critical flow'.

→ For critical flow, $f_r = 1$

$$f_r = \frac{V_c}{\sqrt{gbc}}$$

* Subcritical flow :-

When the depth of flow in a channel is given then the critical depth (h_c) the flow is said to be subcritical flow.

→ For this type of flow, Froude no is less than 1.

$$Fr < 1$$

* Super Critical / shooting flow / Tormented flow :-

When the depth of flow in a channel is less than the critical depth (h_c), then the flow is said to be "super" critical flow.

- For this type of flow Froude no is greater than 1.

$$Fr > 1$$

* Alternating depths :-

In the specific energy curve the points corresponding to the min. specific energy and the depth of flow at 'c' are equal called "critical" depth.

For any other value of this specific energy there are 2 depths.

- 1) Greater than the h_c (critical depth).
- 2) Smaller than h_c (critical depth).

These 2 depths for a given specific energy called the alternate depths.

These depths are shown as h_1 & h_2 .

* Computation of uniform flow :-

The discharge of uniform flow in a channel may be expressed by using Chezy's formula as :

$$Q = AV \quad V = C\sqrt{mi}$$

$$Q = AC\sqrt{mi}$$

$$Q = k\sqrt{T} \rightarrow ①$$

$$k = AC\sqrt{mi} \rightarrow ②$$

18)

The term k is known as Conveyance of the channel section, which is measure of its carrying capacity of the channel section.

- Since, it is directly proportional to the discharge (Q).

- Similarly when the manning's formula is used the discharge becomes.

$$Q = k \sqrt{I} \quad \text{--- (3)}$$

$$k = \frac{1}{n} A m^{2/3} \quad \text{--- (4)}$$

from eqn (1) & (3).

$$\boxed{k = \frac{Q}{\sqrt{I}}} \quad \text{--- (5).}$$

Eqn (5) can be used to compute the Conveyance when the discharge & bottom slope of the channel are given. On the other hand eqn (2) & (4) are used to compute the Conveyance, when the geometry of the wetted area of the channel section and Chezy's 'C' or Manning's 'n' are given.

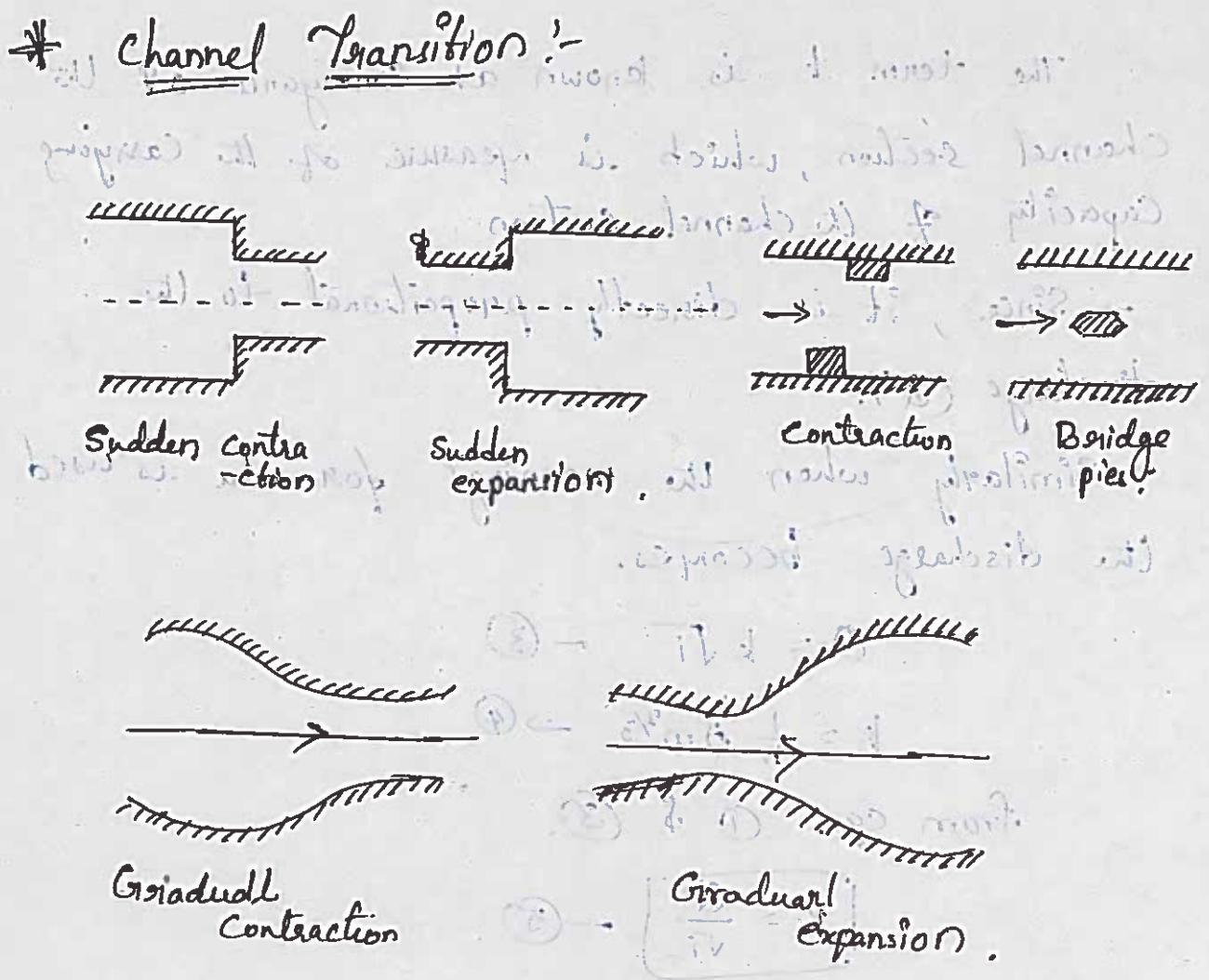
→ Since Manning's formula is used extensively, the expression $A m^{2/3}$ is called the 'section factor'

for uniform flow computation

$$A m^{2/3} = n k$$

$$A m^{2/3} = n \times \frac{Q}{\sqrt{I}} \quad \rightarrow (6) \quad (\because k = \frac{Q}{\sqrt{I}})$$

Eqn (6) quite useful for computation and analysis for uniform flow.



In the case of long channels obtain it becomes necessary to provide transition. A transition is the position of a channel with varying cross-section, which may or may not have the same c/s form.

The variation of a channel section may be caused either by reducing or increasing the width or by raising or lowering the bottom of the channel.

Variants / Various channel transitions may be broadly classified as sudden transition and gradually transitions.

→ Sudden transition are those in which the change of c/s dimensions occurring is a relatively short length on the other hand in case of gradually transition the change of c/s area takes place gradually in a relatively long length of the channel.